Bits, Bytes and Integers

Introduction to Computer Systems

Danfeng Shan
Xi’an Jiaotong University

Announcements

ICSServer已准备好，Tutorial时间暂定周日上午09:30

Lab 1 (Data Lab) 已经公布，截止时间：03.15（周五）

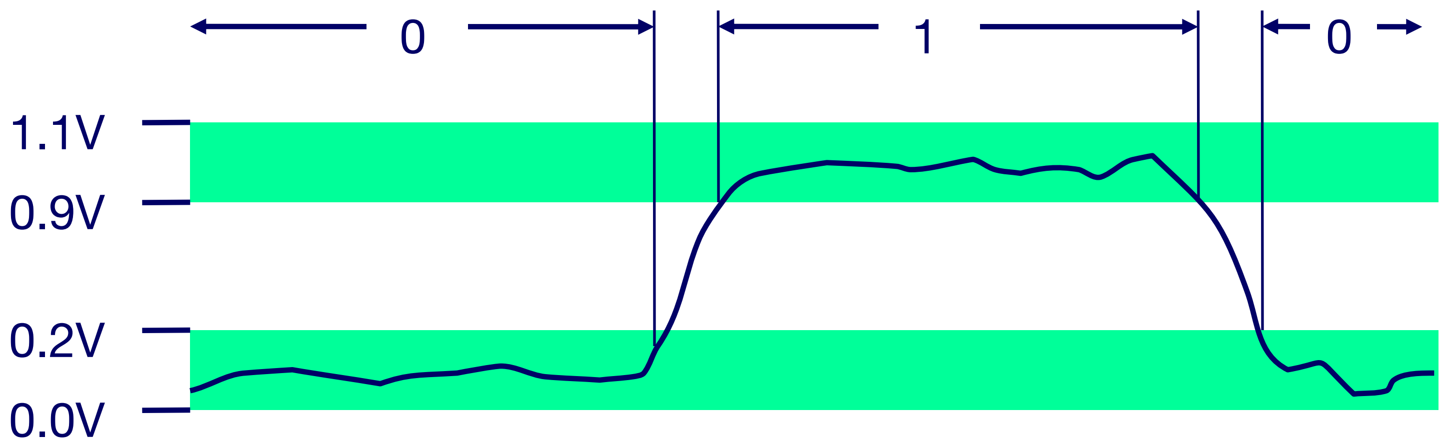
实验课：3月9日（周六）上午8:00-12:00，西一楼实验中心机房

Today: Bits, Bytes, and Integers

- **Representing information as bits**
- **Bit-level manipulations**
- **Integers**
  - Representation: unsigned and signed
  - Conversion, casting
  - Expanding, truncating
  - Addition, multiplication, shifting
- **Representations in memory, pointers, strings**

Everything is bits

- **Each bit is 0 or 1**
- **By encoding/interpreting sets of bits in various ways**
  - Computers determine what to do (instructions)
  - … and represent and manipulate numbers, sets, strings, etc…
- **Why bits? Electronic Implementation**
  - Easy to store with bistable elements (双稳态器件)
  - Reliably transmitted on noisy and inaccurate wires



For example, can count in binary

Base 2 Number Representation

Represent 15213_{10} as 11101101101101_2

Represent 1.20_{10} as $1.0011001100110011[0011]\ldots_2$

Represent 1.5213×10^4 as $1.1101101101101_2 \times 2^{13}$

Encoding Byte Values

Byte = 8 bits

Binary 00000000_2 to 11111111_2

Decimal: 0_{10} to 255_{10}

Hexadecimal 00_{16} to FF_{16}

- Base 16 number representation
- Use characters '0' to '9' and 'A' to 'F'
- Write $FA1D37B_{16}$ in C as
  - 0xFA1D37B
  - 0xfa1d37b

Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

$\texttt{15213:}\ \underbrace{\texttt{0011}}_{\texttt{3}}\ \underbrace{\texttt{1011}}_{\texttt{B}}\ \underbrace{\texttt{0110}}_{\texttt{6}}\ \underbrace{\texttt{1101}}_{\texttt{D}}$

Example Data Representations

C Data Type	Typical 32-bit	Typical 64-bit
`char`	1	1
`short`	2	2
`int`	4	4
`long`	4	8
`float`	4	4
`double`	8	8
pointer	4	8

Example Data Representations

C Data Type	Typical 32-bit	Typical 64-bit
`char`	1	1
`short`	2	2
`int`	4	4
`long`	4	8
`float`	4	4
`double`	8	8
pointer	4	8

Example Data Representations

C Data Type	Typical 32-bit	Typical 64-bit
`char`	1	1
`short`	2	2
`int`	4	4
`long`	4	8
`float`	4	4
`double`	8	8
pointer	4	8
	"ILP32"	"LP64"

Today: Bits, Bytes, and Integers

- Representing information as bits
- **Bit-level manipulations**
- Integers
  - Representation: unsigned and signed
  - Conversion, casting
  - Expanding, truncating
  - Addition, multiplication, shifting
- Representations in memory, pointers, strings

Boolean Algebra

Developed by George Boole in 19th Century

Algebraic representation of logic

Encode "True" as 1 and "False" as 0

And

A&B = 1 when both A=1 and B=1

&	0	1
0	0	0
1	0	1

Or

A|B = 1 when either A=1 or B=1 or both

|	0	1
0	0	1
1	1	1

Not

~A = 1 when A=0

~	0	1
	1	0

Exclusive-Or (Xor)

A^B = 1 when A=1 or B=1, but not both

^	0	1
0	0	1
1	1	0

General Boolean Algebras

Operate on Bit Vectors

Operations applied bitwise

```
  01101001     01101001     01101001
& 01010101   | 01010101   ^ 01010101   ~ 01010101
  01000001     01111101     00111100     10101010
```

Bit-Level Operations in C

Operations &, |, ~, ^ Available in C

Apply to any "integral" data type

`long, int, short, char, unsigned`

View arguments as bit vectors

Arguments applied bit-wise

Contrast: Logic Operations in C

Contrast to Bit-Level Operators

Logic Operations: &&, ||, !

- View 0 as "False"
- Anything nonzero as "True"
- Always return 0 or 1
- Early termination

Examples (char data type)

```
!0x41 →   0x00
!0x00 →   0x01
!!0x41→   0x01

0x69 && 0x55 →   0x01
0x69 || 0x55 →   0x01
p && *p
```

(avoids null pointer access)

Watch out for && vs. & (and || vs. |)…
Super common C programming pitfall!

Shift Operations

Left Shift: `x << y`

Shift bit-vector **x** left **y** positions

– Throw away extra bits on left

Fill with 0's on right

Right Shift: `x >> y`

Shift bit-vector **x** right **y** positions

Throw away extra bits on right

Logical shift

Fill with 0's on left

Arithmetic shift

Replicate most significant bit on left

Undefined Behavior

Shift amount < 0 or ≥ word size

Argument **x**	01100010
<< 3	00010*000*
Log. >> 2	*00*011000
Arith. >> 2	*00*011000

Argument **x**	10100010
<< 3	00010*000*
Log. >> 2	*00*101000
Arith. >> 2	*11*101000

Today: Bits, Bytes, and Integers

- Representing information as bits
- Bit-level manipulations
- Integers
  - Representation: unsigned and signed, negation
  - Conversion, casting
  - Expanding, truncating
  - Addition, multiplication, shifting
- Representations in memory, pointers, strings

Question?

```
int foo = -1;
unsigned bar = 1;

(foo < bar) == true ?
```

Encoding "Integers"

Unsigned

Given a bit vector x, w bits long…

$$\mathrm{B2U}(x) = \sum_{i=0}^{w-1} x_i \cdot 2^i$$

Signed (twos complement)

$$\mathrm{B2T}(x) = -x_{w-1} \cdot 2^{w-1} + \sum_{i=0}^{w-2} x_i \cdot 2^i$$

Sign Bit

Examples (w = 5)

±16	8	4	2	1
0	1	0	1	0

$0 + 8 + 0 + 2 + 0 = 10$

16	8	4	2	1
1	0	1	1	0
-16	8	4	2	1

$16 + 8 + 0 + 2 + 0 = 26$

$-16 + 8 + 0 + 2 + 0 = -10$

Negation: Complement & Increment

Negate through complement and increase

`~x + 1 == -x`

Why?

```
-x + x       == 0  (by definition)
~x + x       == 1111…111 == -1
~x + x + 1   == 0
(~x+1) + x   == 0
 ~x+1        == -x
```

	x	1 0 0 1 1 1 0 1
+	~x	0 1 1 0 0 0 1 0
	–1	1 1 1 1 1 1 1 1

Example: x = 15213

	Decimal	Hex	Binary
`x`	**15213**	`3B 6D`	`00111011 01101101`
`~x`	**-15214**	`C4 92`	`11000100 10010010`
`~x+1`	**-15213**	`C4 93`	`11000100 10010011`
`y`	**-15213**	`C4 93`	`11000100 10010011`

Complement & Increment Examples

$x = 0$

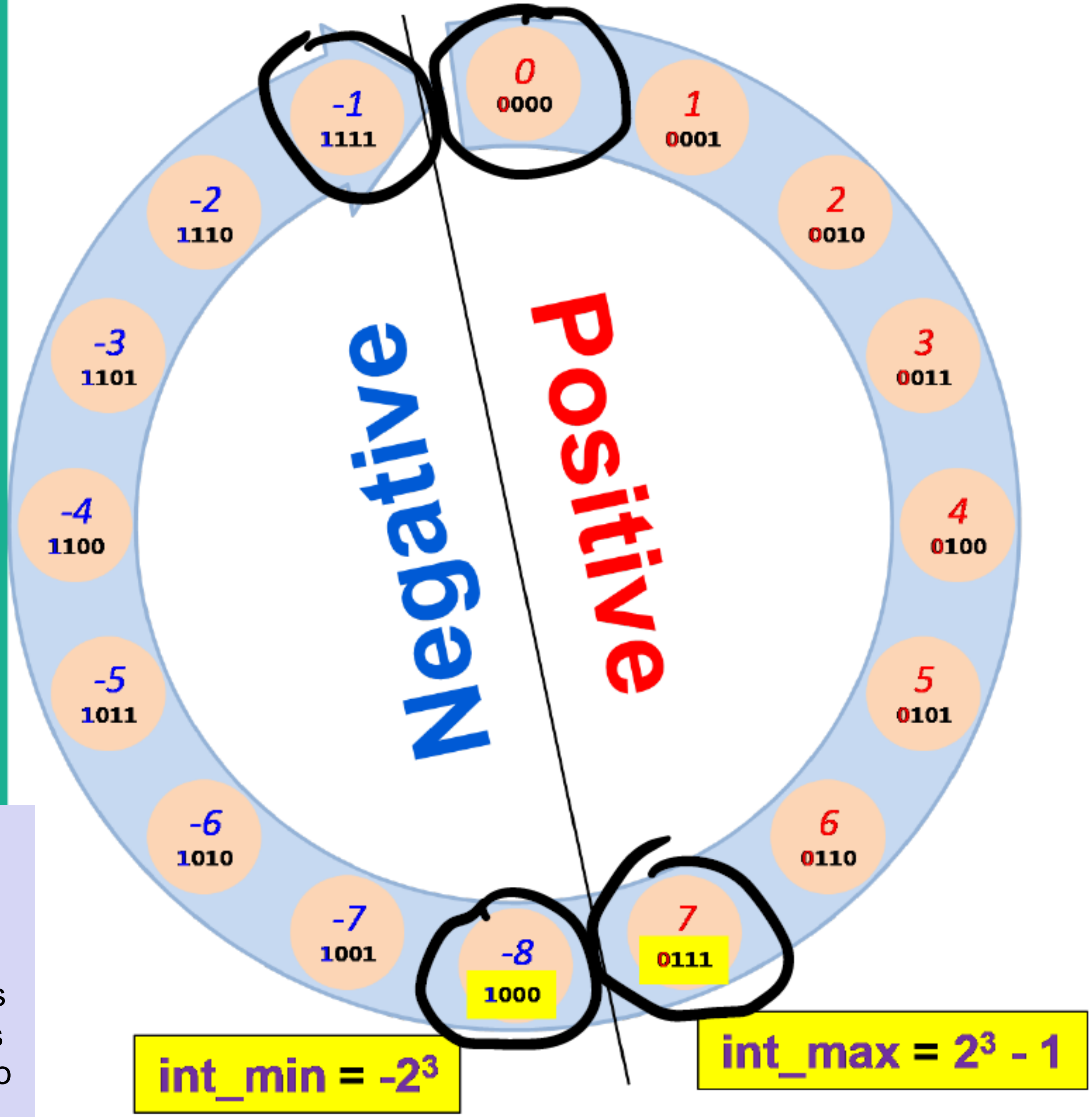
	Decimal	Hex	Binary
0	**0**	00 00	00000000 00000000
~0	**-1**	FF FF	11111111 11111111
~0+1	**0**	00 00	00000000 00000000

$x = T_{\min}$

	Decimal	Hex	Binary
x	**-32768**	80 00	10000000 00000000
~x	**32767**	7F FF	01111111 11111111
~x+1	**-32768**	80 00	10000000 00000000

Oops!
It's still
negative!

Eight negative values:
−1, − 2, …, − 8



Eight *non*-negative values:
0, 1, …, 7

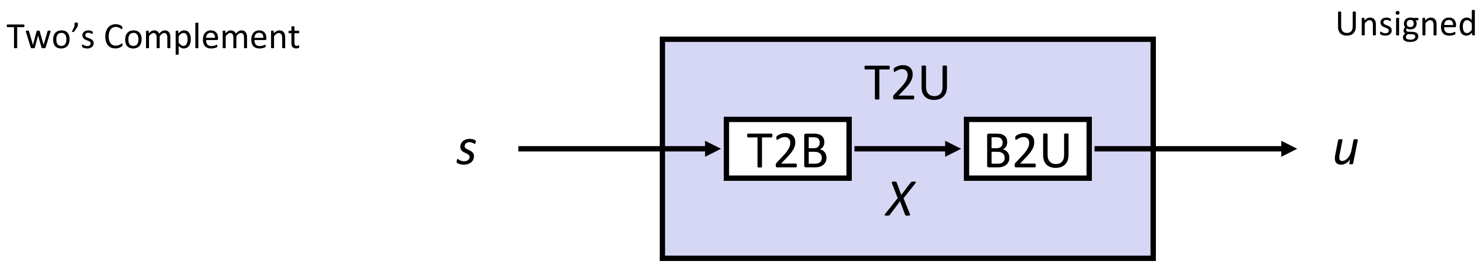
Mathematicians would prefer it if a 4-bit signed number could represent values −8…8, but that's $2^4 + 1$ values, so they won't all fit.

What if we made a 4-bit signed number only represent values −7…7? Then we wouldn't be using bit pattern `1000`…

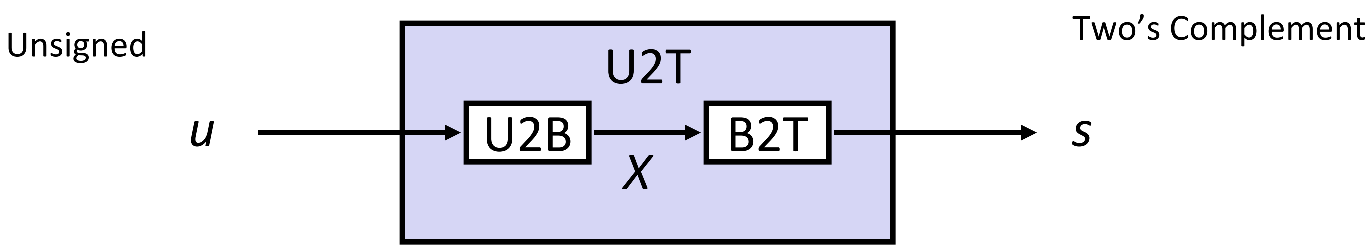
Today: Bits, Bytes, and Integers

- Representing information as bits
- Bit-level manipulations
- Integers
  - Representation: unsigned and signed, negation
  - Conversion, casting
  - Expanding, truncating
  - Addition, multiplication, shifting
- Representations in memory, pointers, strings

Mapping Between Signed & Unsigned



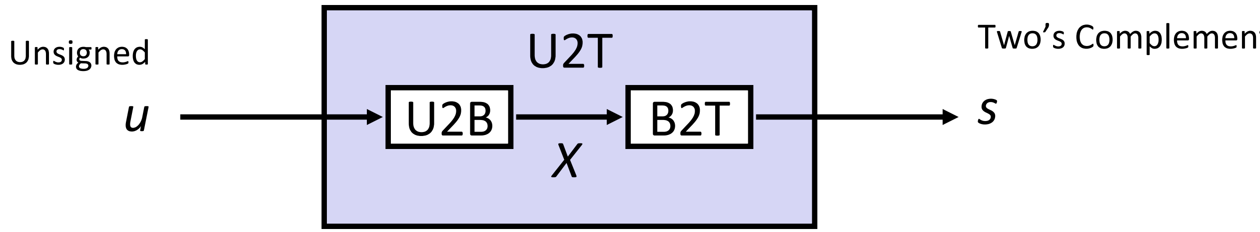
Maintain Same Bit Pattern



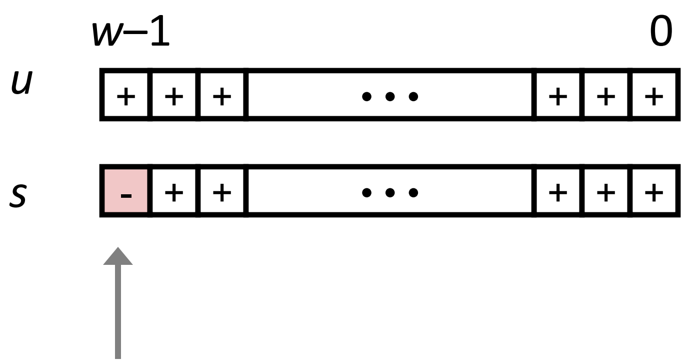
Maintain Same Bit Pattern

Mappings between unsigned and two's complement numbers:
Keep bit representations and reinterpret

Relation between Signed & Unsigned



Maintain Same Bit Pattern



Large positive weight
becomes
Large negative weight

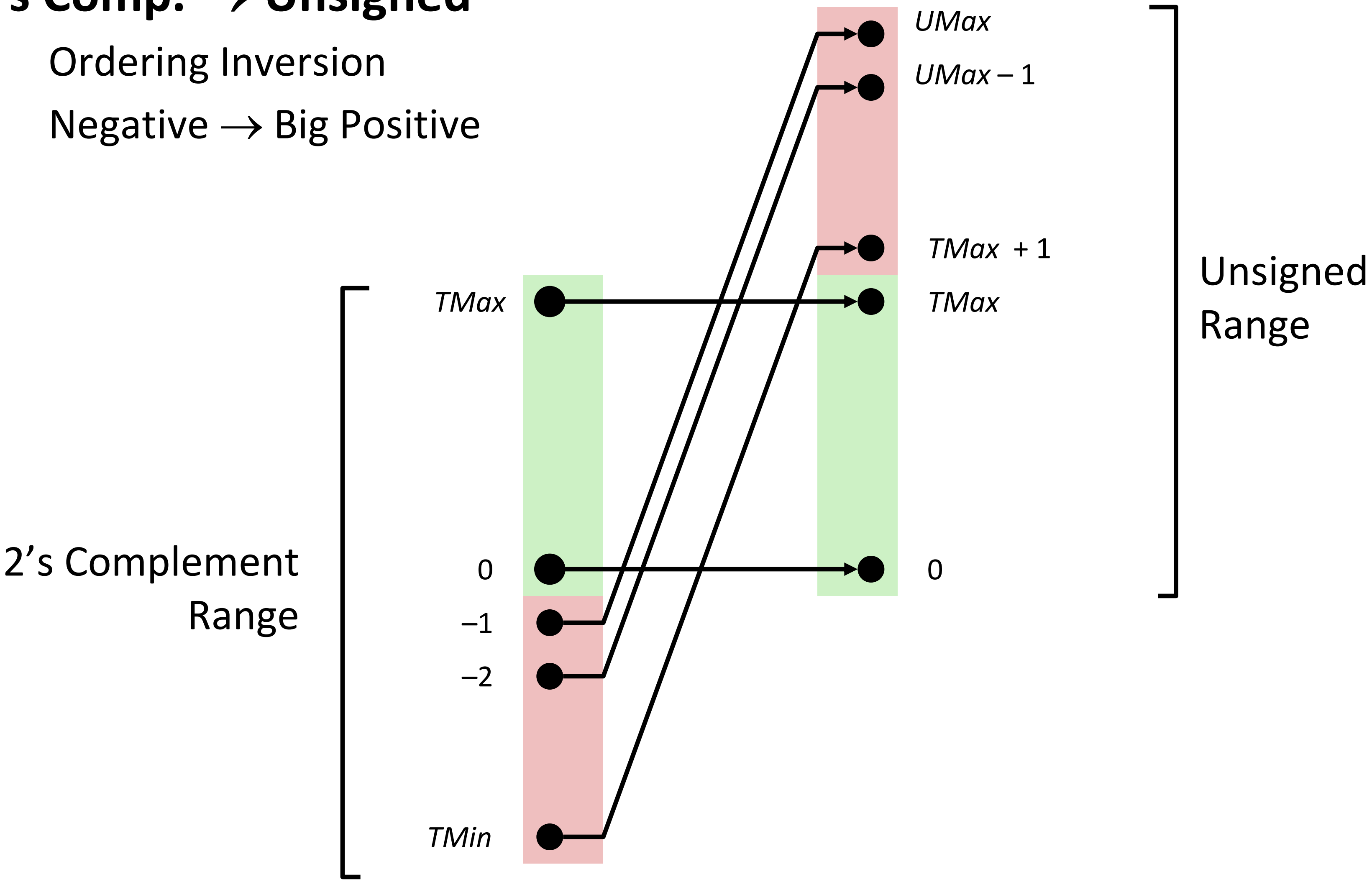
Mapping Signed ↔ Unsigned

Bits	Signed		Unsigned
0000	0		0
0001	1		1
0010	2		2
0011	3	=	3
0100	4		4
0101	5		5
0110	6		6
0111	7		7
1000	-8		8
1001	-7		9
1010	-6	± 16	10
1011	-5		11
1100	-4		12
1101	-3		13
1110	-2		14
1111	-1		15

Conversion Visualized

2's Comp. → Unsigned

- Ordering Inversion
- Negative → Big Positive



Signed vs. Unsigned in C

Constants

By default are considered to be signed integers

Unsigned if have "U" as suffix

0U, 4294967259U

Casting

Explicit casting between signed & unsigned same as U2T and T2U

```
int tx, ty;
unsigned ux, uy;
tx = (int) ux;
uy = (unsigned) ty;
```

Implicit casting also occurs via assignments and procedure calls

```
tx = ux;
uy = ty;
```

```
int fun(unsigned u);
uy = fun(tx);
```

Casting Surprises

Expression Evaluation

If there is a mix of unsigned and signed in single expression, ***signed values implicitly cast to unsigned***

Including comparison operations **<, >, ==, <=, >=**

Examples:

Constant 1	Constant 2	Relation	Evaluation
0	0U	==	Unsigned
-1	0	<	Signed
-1	0U	>	Unsigned
INT_MAX	INT_MIN	>	Signed
(unsigned)INT_MAX	INT_MIN	<	Unsigned
-1	-2	>	Signed
(unsigned)-1	-2	>	Unsigned
INT_MAX	((unsigned)INT_MAX) + 1	<	Unsigned
INT_MAX	(int)(((unsigned)INT_MAX) + 1)	>	Signed

Question?

example02.c

```
int foo = -1;
unsigned bar = 1;

foo < bar == true ?
```

Summary
Casting Signed ↔ Unsigned: Basic Rules

- Bit pattern is maintained
- But reinterpreted
- Can have unexpected effects: adding or subtracting 2^w

Expression containing signed and unsigned int

- `int` is cast to `unsigned`!!

Today: Bits, Bytes, and Integers

- **Representing information as bits**
- **Bit-level manipulations**
- **Integers**
  - Representation: unsigned and signed, negation
  - Conversion, casting
  - Expanding, truncating
  - Addition, multiplication, shifting
- **Representations in memory, pointers, strings**

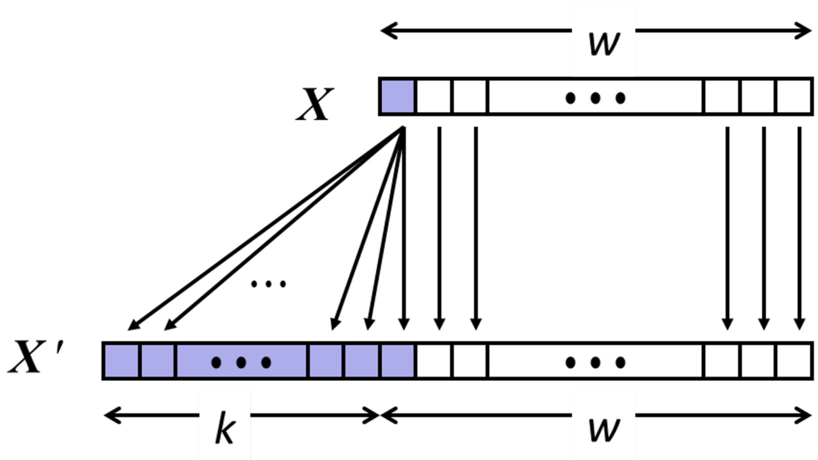
Question?

example03.c

```
int x = 0x8000;
short sx = (short) x;
int y = sx;
```

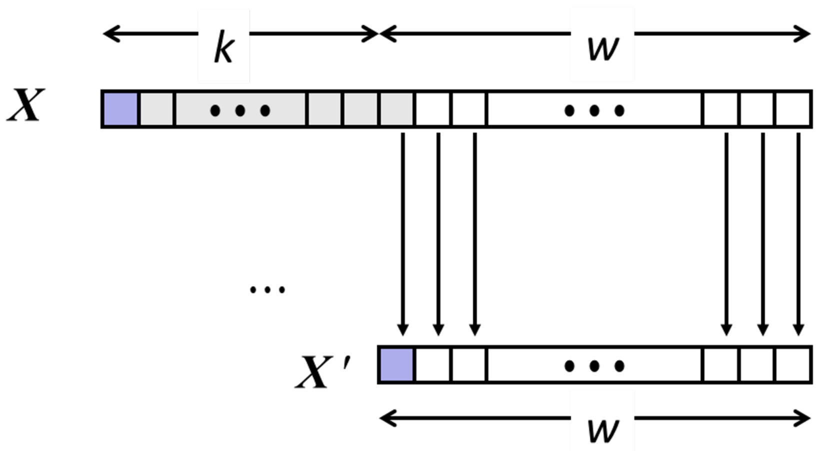
Sign Extension and Truncation

Sign Extension



Make k copies of sign bit

Truncation



Chop off k highest bits

Sign Extension: Simple Example

Positive number

	-16	8	4	2	1
10 =	0	1	0	1	0

	-32	16	8	4	2	1
10 =	0	0	1	0	1	0

Negative number

	-16	8	4	2	1
-10 =	1	0	1	1	0

	-32	16	8	4	2	1
-10 =	1	1	0	1	1	0

Truncation: Simple Example

No sign change

	-16	8	4	2	1
2 =	0	0	0	1	0

	-8	4	2	1
2 =	0	0	1	0

	-16	8	4	2	1
-6 =	1	1	0	1	0

	-8	4	2	1
-6 =	1	0	1	0

Sign change

	-16	8	4	2	1
10 =	0	1	0	1	0

	-8	4	2	1
-6 =	1	0	1	0

	-16	8	4	2	1
-10 =	1	0	1	1	0

	-8	4	2	1
6 =	0	1	1	0

Question?

example03.c

```c
int x = 0x8000;
short sx = (short) x;
int y = sx;
```

Today: Bits, Bytes, and Integers

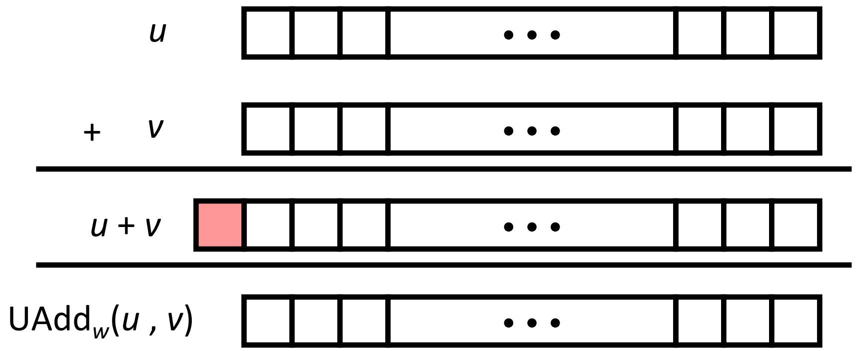
- **Representing information as bits**
- **Bit-level manipulations**
- **Integers**
  - Representation: unsigned and signed
  - Conversion, casting
  - Expanding, truncating
  - Addition, multiplication, shifting
- **Representations in memory, pointers, strings**

Unsigned Addition

Operands: w bits

True Sum: w+1 bits

Discard Carry: w bits



Standard Addition Function

Ignores carry output

```
unsigned char      1110 1001        E9        233
                 +  1101 0101     + D5      + 213
                 ------------     ----      -----
                 ------------     ----      -----
```

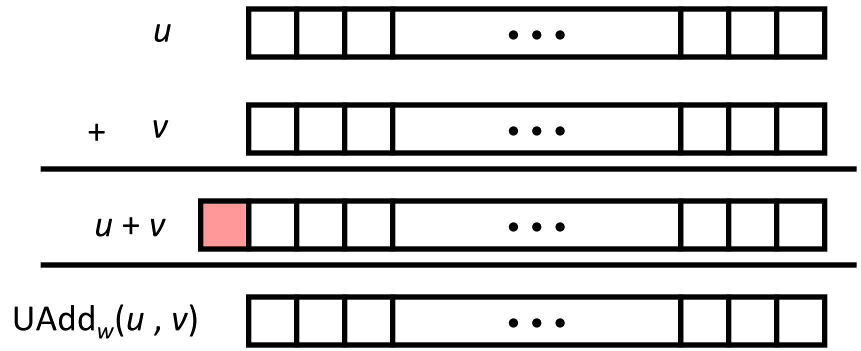
Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

Unsigned Addition

Operands: *w* bits

True Sum: *w*+1 bits

Discard Carry: *w* bits



Standard Addition Function

Ignores carry output

unsigned char			
	1110 1001	E9	233
	+ 1101 0101	+ D5	+ 213
	1 1011 1110	1BE	446
	1011 1110	BE	190

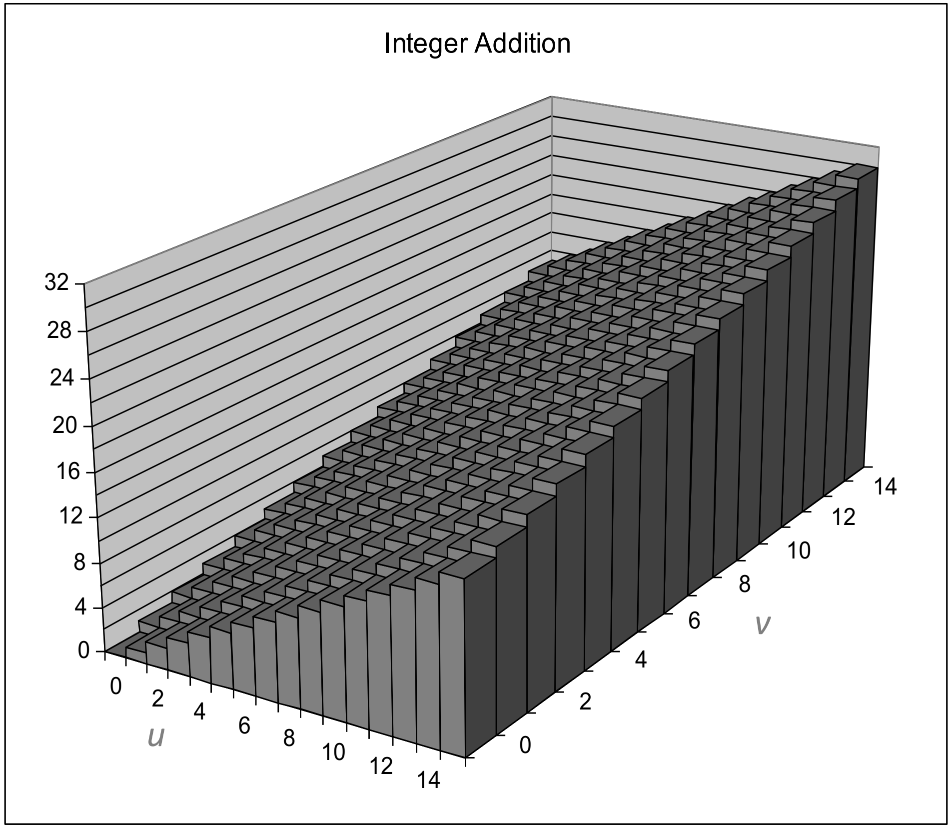
Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

Visualizing (Mathematical) Integer Addition

Integer Addition

- 4-bit integers *u*, *v*
- Compute true sum $\text{Add}_4(u, v)$
- Values increase linearly with *u* and *v*
- Forms planar surface

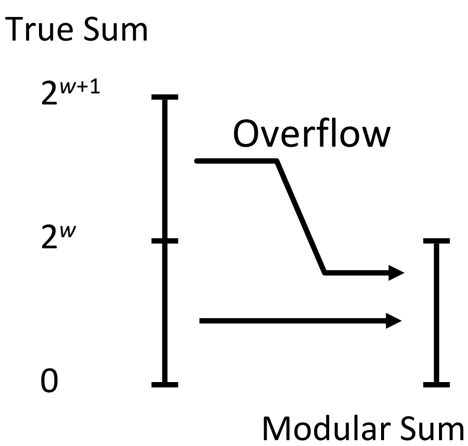
$\text{Add}_4(u, v)$

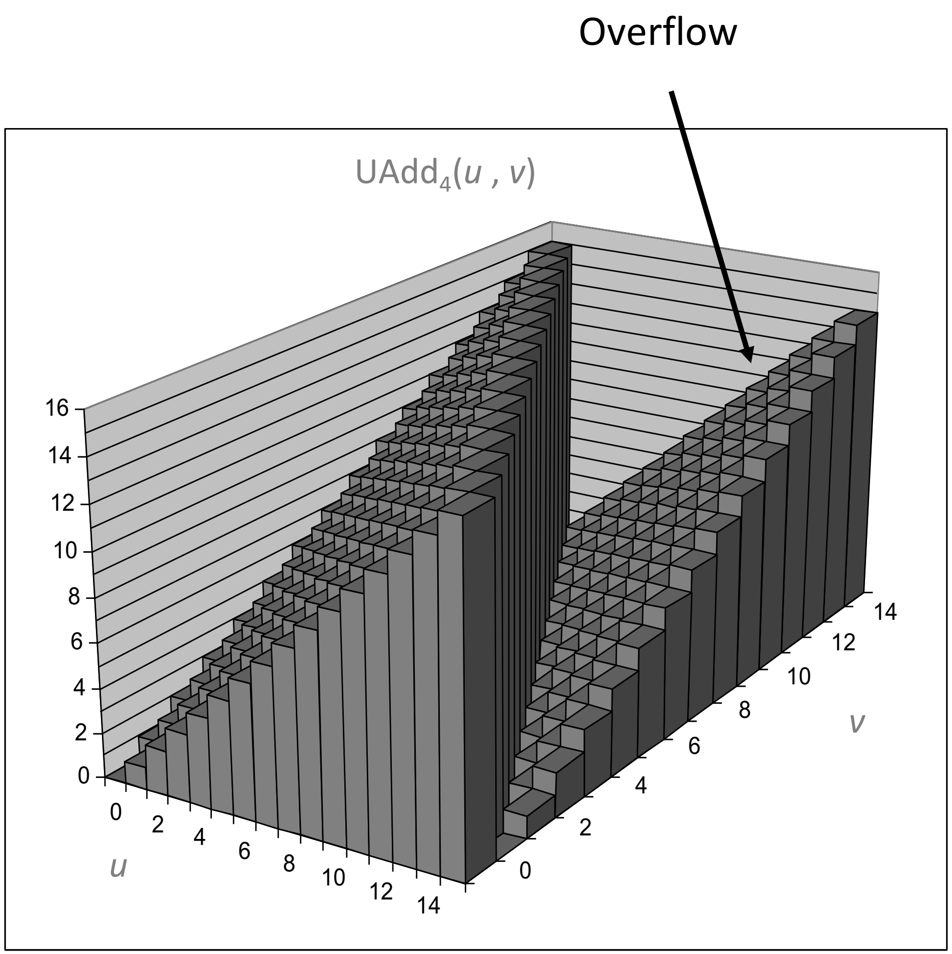


Visualizing Unsigned Addition

Wraps Around

- If true sum ≥ 2^w
- At most once



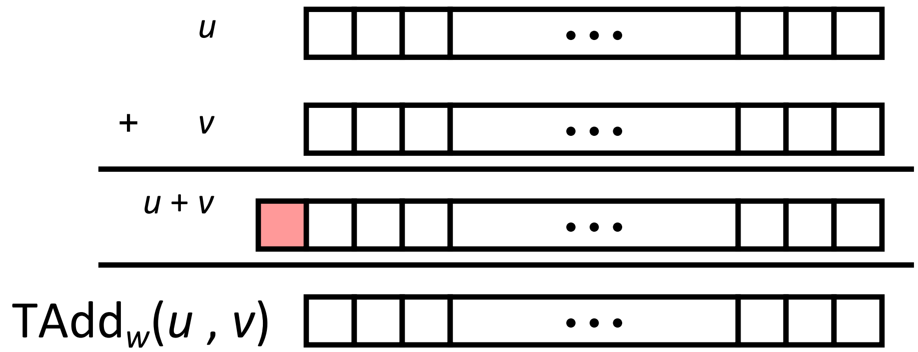


Two’s Complement Addition

Operands: w bits

True Sum: w+1 bits

Discard Carry: w bits



TAdd and UAdd have Identical Bit-Level Behavior

Signed vs. unsigned addition in C:

```
int s, t, u, v;
s = (int) ((unsigned) u + (unsigned) v);
t = u + v
```

Will give `s == t`

1110 1001	E9	-23
+ 1101 0101	+ D5	+ -43
1 1011 1110	1BE	-66
1011 1110	BE	-66

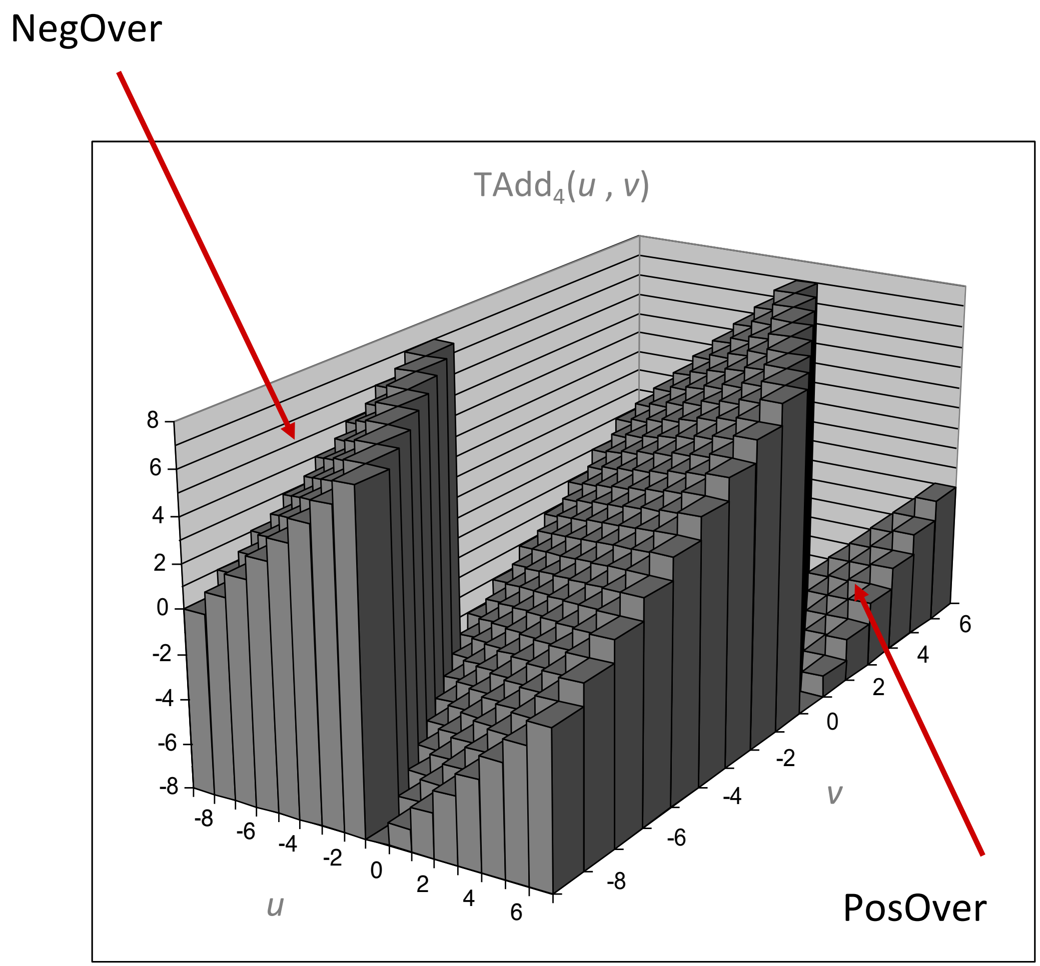
Visualizing 2's Complement Addition

Values

- 4-bit two's comp.
- Range from -8 to +7

Wraps Around

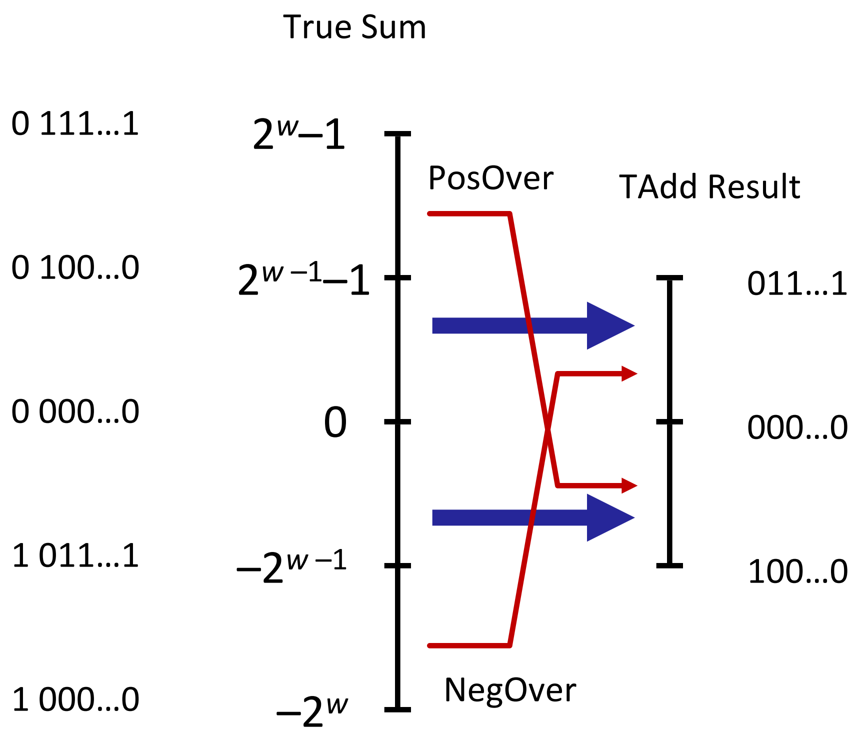
- If sum $\geq 2^{w-1}$
  - Becomes negative
  - At most once
- If sum $< -2^{w-1}$
  - Becomes positive
  - At most once



TAdd Overflow

Functionality

- True sum requires w+1 bits
- Drop off MSB
- Treat remaining bits as 2's comp. integer



Today: Bits, Bytes, and Integers

- **Representing information as bits**
- **Bit-level manipulations**
- **Integers**
  - Representation: unsigned and signed
  - Conversion, casting
  - Expanding, truncating
  - Addition, multiplication, shifting
- **Representations in memory, pointers, strings**

Shifting

Left Shift: `x << y`

- Shift bit-vector `x` left `y` positions
  - Throw away extra bits on left
- Fill with 0's on right
- Equivalent to multiplying by 2^y

Right Shift: `x >> y`

- Shift bit-vector `x` right `y` positions
  - Throw away extra bits on right
- Two kinds:
  - "Logical": Fill with 0's on left
  - "Arithmetic": Replicate most significant bit on left
- *Almost* equivalent to dividing by 2^y

Undefined Behavior (in C)

- Shift amount < 0 or ≥ word size

Argument x	01100010
<< 3	00010000
Logical >> 2	00011000
Arithmetic >> 2	00011000

Argument x	10100010
<< 3	00010000
Logical >> 2	00101000
Arithmetic >> 2	11101000

Multiplication

Goal: Computing Product of *w*-bit numbers *x*, *y*

Either signed or unsigned

But, exact results can be bigger than *w* bits

Unsigned: up to 2*w* bits

Result range: $0 \le x * y \le (2^{w} - 1)^{2} = 2^{2w} - 2^{w+1} + 1$

Two's complement min (negative): Up to 2*w*-1 bits

Result range: $x * y \ge (-2^{w-1})*(2^{w-1}-1) = -2^{2w-2} + 2^{w-1}$

Two's complement max (positive): Up to 2*w* bits, but only for $(TMin_w)^2$

Result range: $x * y \le (-2^{w-1})^{2} = 2^{2w-2}$

So, maintaining exact results…

would need to keep expanding word size with each product computed

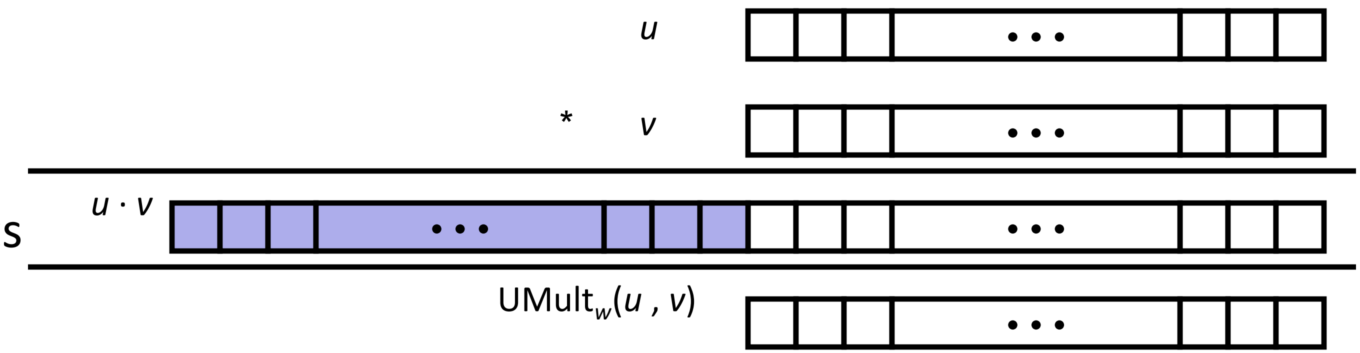
is done in software, if needed

Unsigned Multiplication in C

Operands: *w* bits

True Product: 2**w* bits

Discard *w* bits: *w* bits

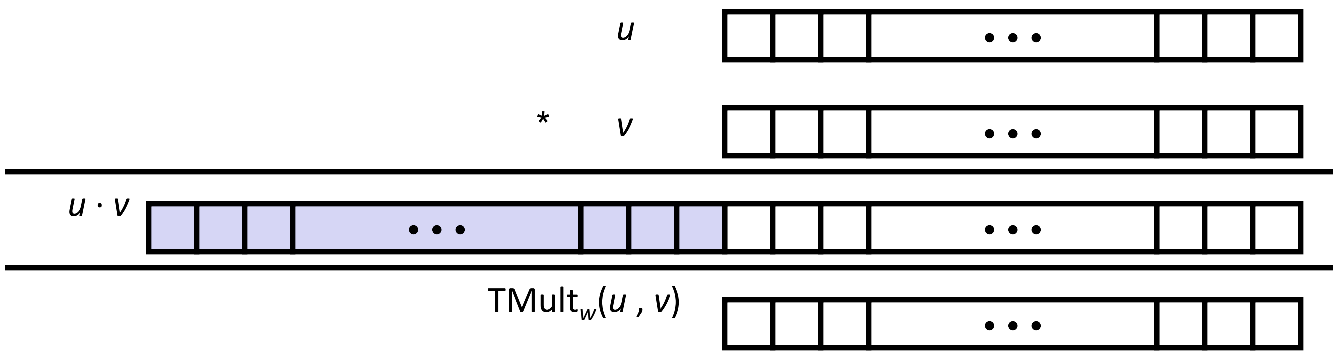


Standard Multiplication Function

Ignores high order *w* bits

```
            1110 1001      E9      233
*           1101 0101   *  D5   *  213
-----------------------------------------
  1100 0001 1101 1101    C1DD    49629
-----------------------------------------
            1101 1101      DD      221
```

Signed Multiplication in C



Operands: *w* bits

True Product: 2**w* bits

Discard *w* bits: *w* bits

Standard Multiplication Function

- Ignores high order *w* bits
- Some of which are different for signed vs. unsigned multiplication
- Lower bits are the same

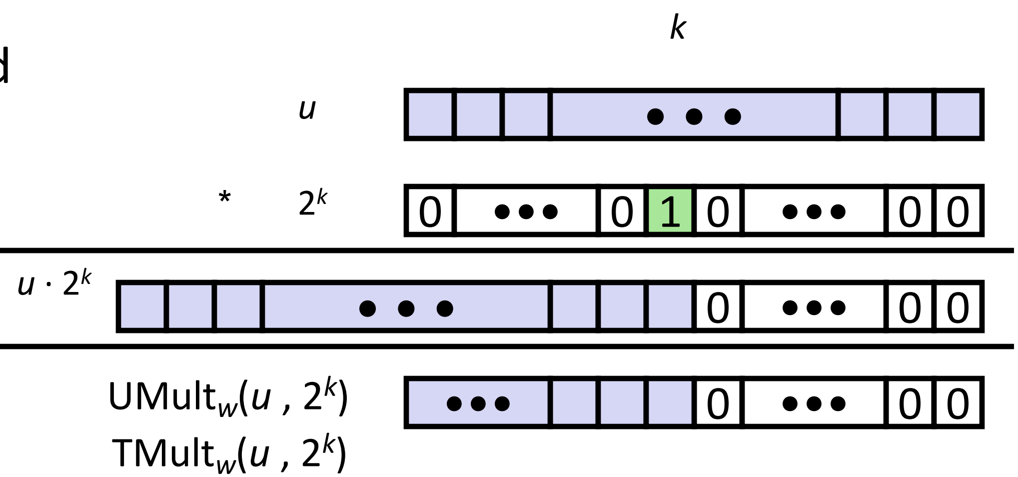
```
            1110 1001        E9          -23
*           1101 0101   *    D5    *     -43
 0000 0011  1101 1101      03DD          989
            1101 1101        DD          -35
```

Power-of-2 Multiply with Shift

Operation

u << k gives **u * 2^k**

Both signed and unsigned



Operands: w bits

True Product: $w+k$ bits

Discard k bits: w bits

Examples

```
u << 3          ==   u * 8
(u << 5) - (u << 3)      ==     u * 24
```

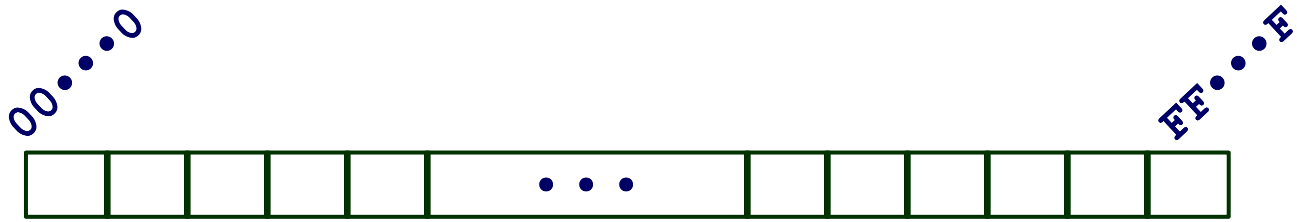
Most machines shift and add faster than multiply

Compiler generates this code automatically

Today: Bits, Bytes, and Integers

- Representing information as bits
- Bit-level manipulations
- Integers
  - Representation: unsigned and signed
  - Conversion, casting
  - Expanding, truncating
  - Addition, multiplication, shifting
- **Representations in memory, pointers, strings**

Byte-Oriented Memory Organization



Programs refer to data by address

- Imagine all of RAM as an enormous array of bytes
- An address is an index into that array
  - A pointer variable stores an address

System provides a private *address space* to each "process"

- A process is an instance of a program, being executed
- An address space is one of those enormous arrays of bytes
- Each program can see only its own code and data within its enormous array
- We'll come back to this later ("virtual memory" classes)

Machine Words

Any given computer has a “Word Size”

Nominal size of integer-valued data

and of addresses

Historically, most machines used 32 bits (4 bytes) as word size

Limits addresses to 4GB (2^{32} bytes)

Recently, machines have 64-bit word size

Potentially, could have 16 EB (exabytes) of addressable memory

That’s 18.4×10^{18} bytes

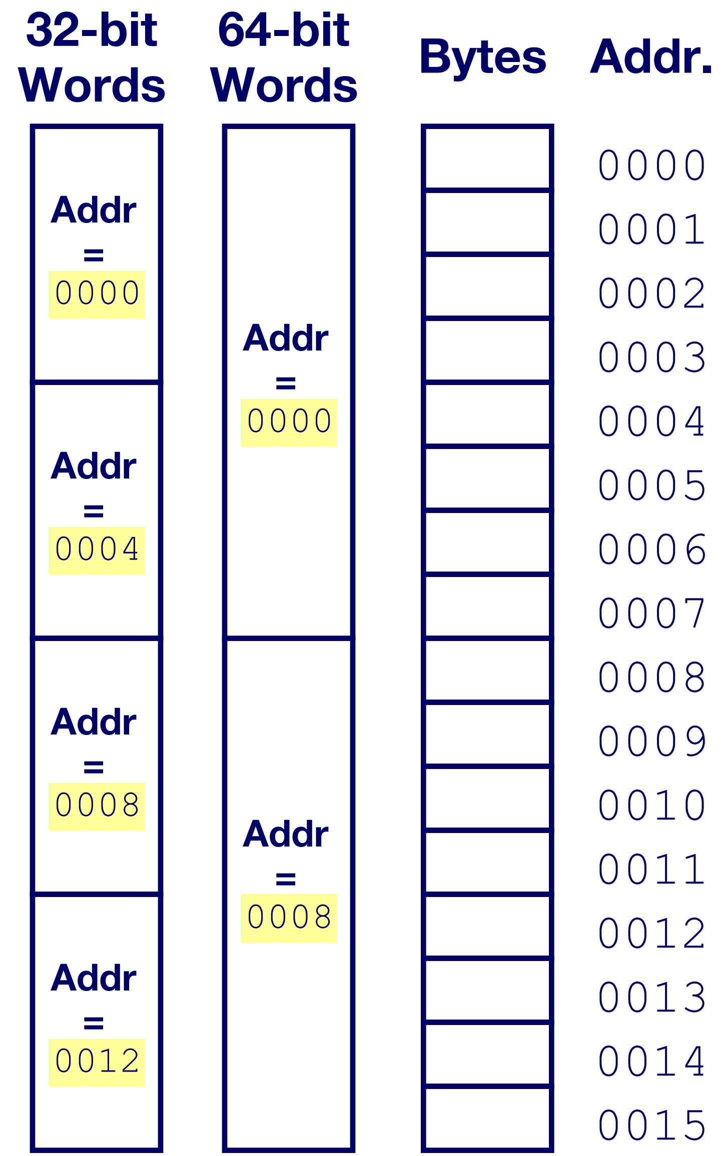
Machines still support multiple data formats

Fractions or multiples of word size

Always integral number of bytes

Addresses *Always* Specify Byte Locations

- Address of a word is address of the first byte in the word
- Addresses of successive words differ by 4 (32-bit) or 8 (64-bit)



Example Data Representations

C Data Type	Typical 32-bit	Typical 64-bit	x86-64
`char`	1	1	1
`short`	2	2	2
`int`	4	4	4
`long`	4	8	8
`float`	4	4	4
`double`	8	8	8
pointer	4	8	8

Question?

example04.c

```
struct foo {
    char mem1[3];  // 3 bytes
    int mem2;      // 4 bytes
    char mem3;     // 1 byte
};
sizeof(struct foo) = ?
```

Byte Ordering

So, how are the bytes within a multi-byte word ordered in memory?

Conventions

Big Endian: Sun, PPC Mac, *network packet headers*

Least significant byte has highest address

Little Endian: *x86*, ARM processors running Android, iOS, and Windows

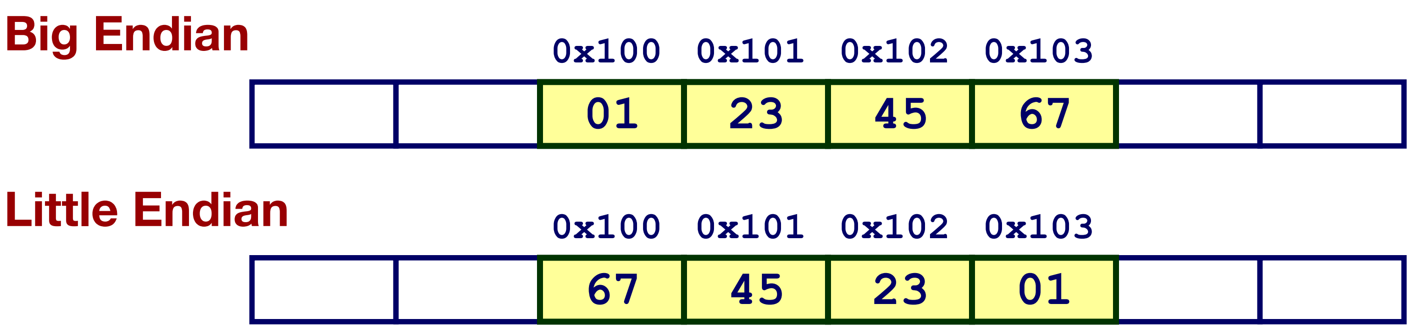
Least significant byte has lowest address

Byte Ordering Example

Example

Variable x has 4-byte value of 0x01234567

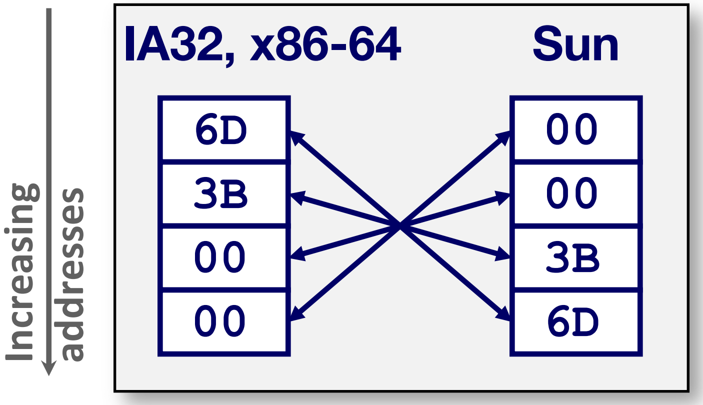
Address given by &x is 0x100



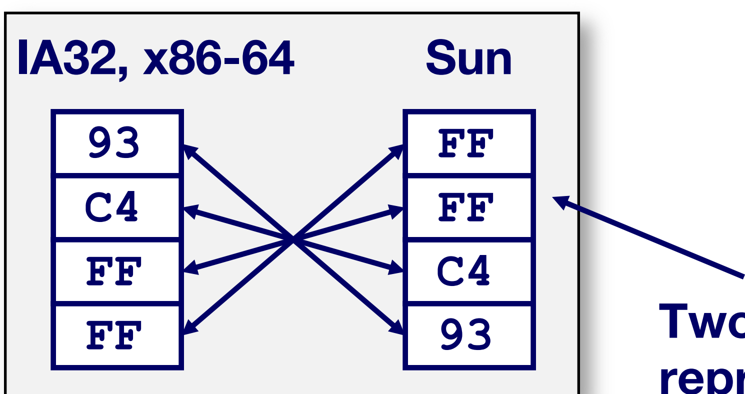
Representing Integers

Decimal:	15213			
Binary:	0011	1011	0110	1101
Hex:	3	B	6	D

`int A = 15213;`

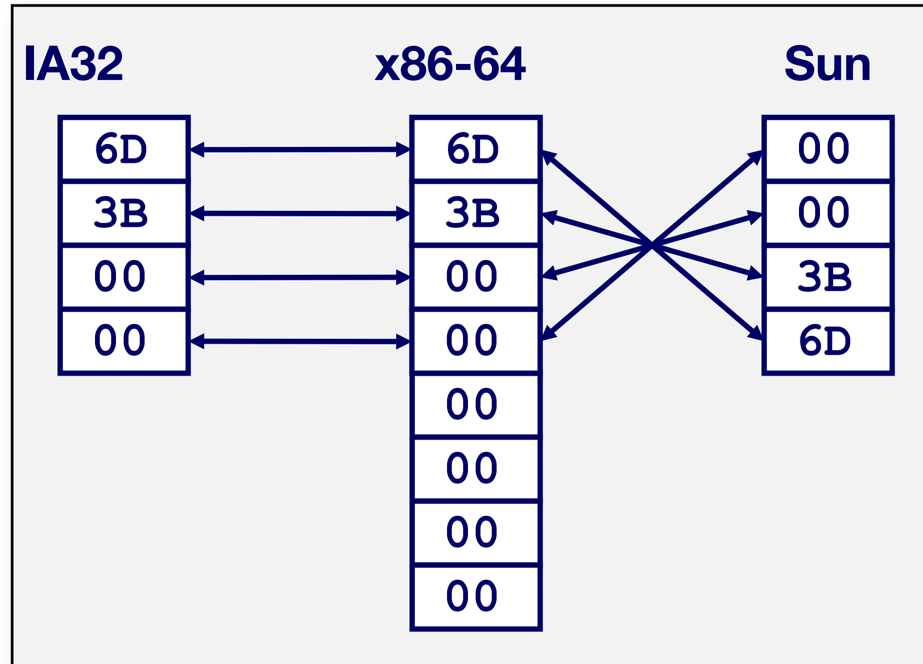


`int B = -15213;`



Two's complement representation

`long int C = 15213;`



Examining Data Representations

Code to Print Byte Representation of Data

Casting pointer to unsigned char * allows treatment as a byte array

```
typedef unsigned char *pointer;

void show_bytes(pointer start, size_t len){
  size_t i;
  for (i = 0; i < len; i++)
    printf(”%p\t0x%.2x\n",start+i, start[i]);
  printf("\n");
}
```

Printf directives:

%p: Print pointer

%x: Print Hexadecimal

`show_bytes` Execution Example

```
int a = 15213;
printf("int a = 15213;\n");
show_bytes((pointer) &a, sizeof(int));
```

Result (Linux x86-64):

```
int a = 15213;
0x7fffb7f71dbc    6d
0x7fffb7f71dbd    3b
0x7fffb7f71dbe    00
0x7fffb7f71dbf    00
```

Representing Pointers

```
int B = -15213;
int *P = &B;
```

Sun	IA32	x86-64
EF	AC	3C
FF	28	1B
FB	F5	FE
2C	FF	82
		FD
		7F
		00
		00

Different compilers & machines assign different locations to objects

Even get different results each time run program

Representing Strings

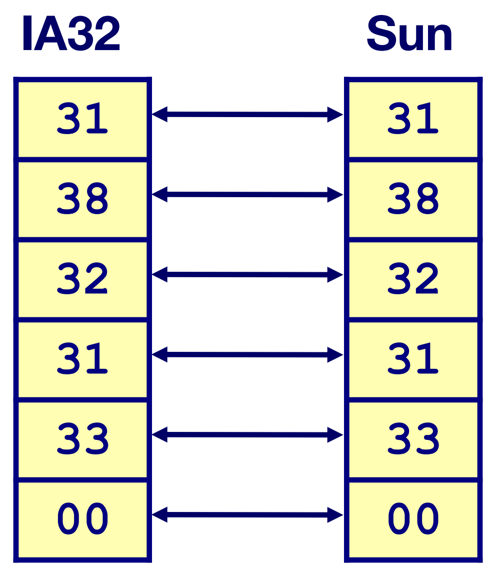
```
char S[6] = "18213";
```

Strings in C

- Represented by array of characters
- Each character encoded in ASCII format
  - Standard 7-bit encoding of character set
  - Character “0” has code 0x30
    - Digit *i* has code 0x30+*i*
- String should be null-terminated
  - Final character = 0

Compatibility

- Byte ordering not an issue



Representing x86 machine code

x86 machine code is a sequence of *bytes*

Grouped into variable-length instructions, which look like strings…

But they contain embedded little-endian numbers…

Example Fragment

Address	Instruction Code	Assembly Rendition
`8048365:`	`5b`	`pop %ebx`
`8048366:`	`81 c3 ab 12 00 00`	`add $0x12ab,%ebx`
`804836c:`	`83 bb 28 00 00 00 00`	`cmpl $0x0,0x28(%ebx)`

Deciphering Numbers

Value:	`0x12ab`
Pad to 32 bits:	`0x000012ab`
Split into bytes:	`00 00 12 ab`
Reverse:	`ab 12 00 00`